

# Topological Defects in $[SU(6)]^3 \times Z_3$

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## Abstract

We study topological defects arising in the Grand Unification Model  $SU(6)_L \otimes SU(6)_c \otimes SU(6)_R \times Z_3$ . We show that the model does not contain domain walls, while it produces massive magnetic monopoles and it may, depending on the symmetry breaking chain, give rise to the formation of strings. We also discuss their possible relation with the origin of the highest energy cosmic rays detected.

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# 1 Introduction

The sources of the high energy cosmic ray (HECR) events recently observed above  $10^{20} \text{ eV}$  ( $100 \text{ EeV}$ ) by the Fly's Eye [1], AGASA [2], Haverah Park [3] and Yakutsk [4] experiments remain unknown. Conventional astrophysical accelerator mechanisms encounter severe difficulties in accelerating particles to these energies [5]. It is hard to accelerate protons and heavy nuclei up to such energies even in the most powerful astrophysical objects. Also, the GZK cut-off [6] limits the distance to a possible source of nucleons with energy above  $\simeq 70 \text{ EeV}$  to less than  $\simeq 100 \text{ Mpc}$  [7] from the Earth. Other possible primary candidates for these HECR events could be gamma rays and neutrinos. Nevertheless, the gamma-ray hypothesis appears inconsistent with the temporal distribution of the Fly's Eye event [8], and also the density profile of the Yakutsk event showed a large number of muons, which argues against this hypothesis [4]. Moreover the mean free path for a  $10^{20} \text{ eV}$  photon before it annihilates on the microwave background into a  $e^-e^+$  pair is around 10 to  $40 \text{ Mpc}$ . On the other hand, the Fly's eye event occurred high in the atmosphere, whereas the expected event rate for early development of a neutrino induced air shower is down from that of an electromagnetic or hadronic interaction by six orders of magnitude [8].

These difficulties have motivated two recent suggestions. The first is that the underlying production mechanism of HECR could be of a non acceleration nature, namely the decay of supermassive elementary particles related to Grand Unified Theories (GUT's). Such particles could be released from topological defects (TD), such as cosmic strings, monopoles and domain walls, which could be formed during the phase transitions associated with the spontaneous symmetry breaking (SB) of the GUT symmetry in the early universe [7, 9, 10]. TD are topologically stable but nevertheless they can release part of their energy in the form of supermassive elementary particles (which could subsequently decay into leptons and quarks) due to physical processes like collapse or annihilation. TD are therefore viable sources of HECR, they predict injection spectra which are considerably harder than shock acceleration spectra and also there is no cut-off effect in the attenuation of ultra high energy  $\gamma$ -rays which dominate the predicted flux [7, 11, 12], although the absolute flux levels predicted by TD are model dependent [13]. The other

suggestion is that the primary particles of the HECR may be relativistic magnetic monopoles, with masses bounded by  $M \leq 10^{10\pm1} \text{ GeV}$ , to be consistent with the Parker limit and other phenomenological bounds [14, 15]. Both suggestions deal with some kind of TD. But in general TD models are very constrained by their astrophysical implications. Supermassive monopoles are perceived to be enough of a cosmological problem [16, 17], so that they have to be “inflated” away [18]. Also they can catalyze proton decay [19]. Conventional GUT’s, where the electroweak and strong forces are unified into a symmetry group, which is broken down to the standard one at an energy scale about  $10^{16} \text{ GeV}$  [20], imply an over abundance of supermassive monopoles formed through the Kibble mechanism [10]. Nevertheless, if one assumes that monopoles exist at the level of abundance compatible with known experimental [21] and phenomenological upper bounds [15], they could give an important contribution to the HECR flux [7, 11].

Domain walls are believed to be cosmological disasters and a particle physics model is considered inadmissible if it predicts them [22]. Superconducting cosmic strings [23] can not produce the HECR flux at the present epoch without violating the  $^4\text{He}$ -photodisintegration bound [24]. Cosmic strings, on the other hand, are considered astrophysically promising [25], and a viable source of HECR.

In this way, TD arising from an specific Grand Unification Model (GUM) come to be of interest. Moreover, the restrictions on TD and their astrophysical implications form a phenomenological test for all the GUM’s proposed until now. Some work on this issue have been done in the most popular models in the past (see for example references [26]). This leads us to study the possible contributions to HECR from TD arising in a new GUT model proposed in recent years [27, 28, 29], which is based on the gauged symmetry  $[SU(6)]^3 \times Z_3$  and which has a set of properties which make it a viable proposal for the symmetry of the non-gravitational interactions in nature. In the present article we develop the first part of this program; here we analyze the SB patterns in the model and establish what kind of TD are involved.

In section 2 we review the  $[SU(6)]^3 \times Z_3$  Grand Unification Model following basically Refs. [27, 28, 29] paying attention to its properties and advantages. In section 3 we discuss the absence of domain walls and cosmic

strings in the model. This occurs even if the SB pattern keeps the discrete symmetry  $Z_3$  intact in the first step. However as we will see, cosmic strings can arise in the model if different symmetry breaking chains are considered. Thus for the symmetry breaking involved the only TD generated in the phase transitions of the model are monopoles and textures. This is discussed in section 4, where we also discuss briefly the possible relation of monopoles with the mechanisms of production of HECDR mentioned in the second paragraph. Finally in section 5 we give some concluding remarks. One appendix at the end of the paper deals with details of the SB implemented in section 3.

## 2 Brief review of $[SU(6)]^3 \times Z_3$

The model under consideration is based on the gauge group

$$G \equiv SU(6)_L \otimes SU(6)_c \otimes SU(6)_R \times Z_3 \quad (2.1)$$

and unifies non-gravitational forces with transitions among three families. In Eq. (2.1)  $\otimes$  indicates a direct product,  $\times$  a semidirect one, and  $Z_3$  is a three-element cyclic group acting upon  $[SU(6)]^3$  such that if  $(A, B, C)$  is a representation of  $[SU(6)]^3$  with  $A$  a representation of the first factor,  $B$  of the second and  $C$  of the third, then  $Z_3(A, B, C) \equiv (A, B, C) \oplus (B, C, A) \oplus (C, A, B)$  is a representation of  $G$ .  $SU(6)_c$  is a vector-like group which includes three quark-like colors and three lepton-like ones, and it includes as a subgroup the  $SU(3)_c \otimes U(1)_{B-L}$  group of the left-right symmetric (LRS) extension of the Standard Model (SM).  $SU(6)_L \otimes SU(6)_R$  includes the  $SU(2)_L \otimes SU(2)_R$  gauge group of the LRS model.

Among the special properties of this model we may recall that its gauge group,  $G$ , is the maximal unifying group for the three families, with left-right symmetry and with (extended) vector color and that it leads to absolute (perturbative) stability of the proton [30]. The quark-lepton symmetry in this model is maximal, since it contains as many leptons colors as quarks colors. Furthermore, all the fermions in the model, including the known ones, belong to a single irreducible representation (irrep) of  $G$ . On the other hand the presence of the horizontal group in  $SU(6)_L \otimes SU(6)_R$  allows the possibility

of obtaining predictions for the fermion mass matrices [28, 29]. Besides its aesthetic appeal, the viability of the model steams from its capacity to match the observed values of the SM couplings constants (see below). For these reasons we consider it of interest to analyze the TD properties of its SB chain.

The 105 gauge fields (GF's) in  $G$  can be divided in two sets: 70 of them belonging to  $SU(6)_L \otimes SU(6)_R$  and 35 being associated with  $SU(6)_c$ . The first set includes  $W_L^\pm$  and  $W_L^0$  (the GF's of the known weak interactions), the GF's associated with  $SU(2)_R$ , the GF's of the horizontal interactions, and the GF's of the nonuniversal charged and neutral interactions. All of them have electrical charges 0 or  $\pm 1$ . The generators of  $SU(6)_{L(R)}$  may be written in a  $SU(2)_{L(R)} \otimes SU(3)_{HL(HR)}$  basis as

$$\sigma_i \otimes I_3/2\sqrt{3}, \quad I_2 \otimes \lambda_\alpha/2\sqrt{2}, \quad \sigma_i \otimes \lambda_\alpha/2\sqrt{2}, \quad (2.2)$$

where  $\sigma_i$  are the  $2 \times 2$  Pauli matrices,  $\lambda_\alpha$  are the  $3 \times 3$  Gell-Mann matrices, and  $I_2$  and  $I_3$  are the  $2 \times 2$  and  $3 \times 3$  identity matrices respectively. The second set includes the eight gluon fields of  $SU(3)_c$ , nine lepto-quarks ( $X_i, Y_i$  and  $Z_i$ ,  $i = 1, 2, 3$ , with electrical charges  $-2/3, 1/3$  and  $-2/3$  respectively), their nine conjugated, six dileptons ( $P_a^\pm, P^0$  and  $\tilde{P}^0$ ,  $a = 1, 2$ , with electrical charges as indicated), plus the GF's associated with diagonal generators in  $SU(6)_c$  which are not taken into account already in  $SU(3)_c$ .

The fermions of the model are in the irrep 108,

$$\psi(108)_L = Z_3 \psi(6, 1, \bar{6})_L = \psi(6, 1, \bar{6})_L \oplus \psi(\bar{6}, 6, 1)_L \oplus \psi(1, \bar{6}, 6)_L, \quad (2.3)$$

with quantum numbers with respect to  $(SU(3)_c, SU(2)_L, U(1)_Y)$  given by

$$\begin{aligned} \psi(\bar{6}, 6, 1)_L &\equiv \psi_a^\alpha : & 3(3, 2, 1/3) \oplus 6(1, 2, -1) \oplus 3(1, 2, 1), \\ \psi(1, \bar{6}, 6)_L &\equiv \psi_\alpha^A : & 3(\bar{3}, 1, -4/3) \oplus 3(\bar{3}, 1, 2/3) \oplus 6(1, 1, 2) \oplus 9(1, 1, 0) \\ & & \oplus 3(1, 1, -2), \\ \psi(6, 1, \bar{6})_L &\equiv \psi_A^a : & 9(1, 2, 1) \oplus 9(1, 2, -1), \end{aligned}$$

where  $a, b, \dots, A, B, \dots, \alpha, \beta, \dots = 1, \dots, 6$  label  $L, R$  and  $C$  tensor indices, respectively. The known fermions are contained in the  $\psi(\bar{6}, 6, 1)_L \oplus \psi(1, \bar{6}, 6)_L$  part of  $\psi(108)$ .

In order to achieve the SB we introduce appropriate Higgs scalars. Using the branching rules

$$\begin{aligned}
SU(6)_{L(R)} &\rightarrow SU(2)_{L(R)} \otimes SU(3)_{HL(R)} \\
6 &\rightarrow (2, 3) \\
15 &\rightarrow (1, 6) \oplus (3, \bar{3}) \\
21 &\rightarrow (1, \bar{3}) \oplus (3, 6)
\end{aligned}$$

and

$$\begin{aligned}
SU(6)_c &\rightarrow SU(3)_c \\
6 &\rightarrow (3) + 3(1) \\
15 &\rightarrow (\bar{3}) + 3(3) + 3(1) \\
21 &\rightarrow (6) + 6(1) + 3(3),
\end{aligned}$$

we can see that the vacuum expectation values (vevs) of a 6 of  $SU(6)_L$  necessarily break  $SU(2)_L$ . However, the  $Z_3$  symmetrized version of a 6,  $\phi(18) = Z_3\phi(6, 1, 1)$ , is not sufficient to give tree-level mass to at least one ordinary fermion. We therefore assume that the last step in the SB chain of  $G$  is due to the vevs of a  $\phi_4 = \phi(108) = Z_3\phi(1, \bar{6}, 6)$  and that these vevs lie only in the electrically neutral directions in the  $SU(6)_L \otimes SU(6)_R$  space. These vevs are also chosen in such a way that the *modified horizontal survival hypothesis*<sup>1</sup> [28, 29] holds. The first steps of the SB chain arise from vevs of Higgs fields of the type  $Z_3\phi(\bar{n}, 1, n)$ , where  $n$  may be 15 or 21.

The SB chain is constrained by the requirement that the evolution of the SM coupling constants from the unification scale to the scale  $M_L$  of the last step of the chain, agrees with the experimental values [31]  $\sin^2\theta_W(M_L) = 0.2315$ ,  $\alpha_{EM}^{-1}(M_L) = 127.9$ ,  $\alpha_3(M_L) = 0.113$  and  $M_L \simeq 10^2 GeV$ . For the renormalization group equations (rge), which govern the evolution of the coupling constants, we adopt the *survival hypothesis*<sup>2</sup> [32] and the *extended*

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<sup>1</sup> The modified survival hypothesis states that, out of the known fermions, only the top quark acquires tree level mass while the mass of the rest of these fermions is of radiative origin.

<sup>2</sup> The survival hypothesis amounts to assume that at every step, where a symmetry  $G'$  is broken to  $G''$  at the scale  $M$ , all fermions whose mass is  $G''$  invariant acquire mass of order  $M$ , with the possible exception of the last step of the SB chain.

*survival hypothesis*<sup>3</sup> [33]. When the symmetry is broken down to the SM group in  $N$  steps at the scales  $M_k$ , the coupling constants satisfy, up to one loop, the rge

$$\alpha_i^{-1}(M_0) = f_i \alpha^{-1} - \sum_{k=0}^{N-1} b_i^k \ln \left( \frac{M_{k+1}}{M_k} \right), \quad (2.4)$$

where  $M_0 = M_L$ ,  $\alpha_i = g_i^2/4\pi$ ,  $i = 1, 2, 3$ , and  $g_i$  are, respectively, the gauge coupling constants of the  $U(1)_Y$ ,  $SU(2)_L$  and  $SU(3)_c$  subgroups of the SM. The factors  $f_i$  are constants and define the relation at the unification scale  $M_N$  between  $g$ , the coupling constant of  $[SU(6)]^3 \times Z_3$ , and  $g_i$ . The numerical values of these factors,  $f_1 = 14/3$ ,  $f_2 = 3$  and  $f_3 = 1$  [28, 29], arise from the normalization conditions adopted for the generators of the corresponding gauge group. In Eq. (2.4)

$$b_i^k = \frac{1}{4\pi} \left\{ \frac{11}{3} C_i^k(\text{vectors}) - \frac{2}{3} C_i^k(\text{Weyl fermions}) - \frac{1}{6} C_i^k(\text{scalars}) \right\}, \quad (2.5)$$

where  $C_i^k(\dots)$  are the index of the representation to which the  $(\dots)$  particles are assigned. For a complex scalar field the value of  $C_i^k(\text{scalars})$  should be doubled. The relationships

$$\alpha_{EM}^{-1} \equiv \alpha_1^{-1} + \alpha_2^{-1} \quad \text{and} \quad \tan^2 \theta_W = \frac{\alpha_1}{\alpha_2}, \quad (2.6)$$

where  $\theta_W$  is the weak mixing angle, hold at all the energy scales and from this expressions and Eq. (2.4) we have straightforwardly

$$\alpha_{EM}^{-1}(M_0) - \frac{23}{3} \alpha_3^{-1}(M_0) = \sum_{k=0}^{N-1} \left( \frac{23}{3} b_3^k - b_1^k - b_2^k \right) \ln \left( \frac{M_{k+1}}{M_k} \right) \quad (2.7)$$

and

$$\sin^2 \theta_W(M_0) = 3\alpha_{EM}(M_0) \left[ \alpha_3^{-1}(M_0) + \sum_{k=0}^{N-1} \left( b_3^k - \frac{1}{3} b_2^k \right) \ln \left( \frac{M_{k+1}}{M_k} \right) \right]. \quad (2.8)$$

Now, when  $N > 1$  (the model with only two mass scales was excluded in Ref. [29, 30] by experimental data), the mass scales and their hierarchy are

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<sup>3</sup>For extended survival hypothesis it is understood the assumption that the mass of all the Higgs scalars of the irreps under  $G'$  to which the scalars that acquire  $G'$  breaking vevs belong, is of order  $M$ . The rest of the scalars that complete an irrep under  $G$  ( $G' \subset G$ ), have bigger masses and are decoupled in the rge below  $M$ .

established by forcing the solutions of the rge to agree with the experimental data. In other words, the evolution of the coupling constants is determined by the values of the intermediate scales,  $M_k$ , and the number of them.

The type of topological defects depends then on the active symmetry at each step. This is the subject of the next sections.

### 3 Strings and domain walls.

Cosmic strings, monopoles, and other topological structures can appear during phase transitions when a gauge group  $\mathcal{G}$  is broken down spontaneously to a subgroup  $\mathcal{H}$  at certain energy scale  $M_X$ . The topological criterion for the existence of a string is the nontriviality of the fundamental homotopy group of the vacua quotient manifold  $\mathcal{M} = \mathcal{G}/\mathcal{H}$ , denoted by  $\pi_1(\mathcal{M}) (\neq 0)$  [16, 17, 34]. Similar to strings, the presence of other TD such as domain walls, monopoles and textures is associated with the existence of a non trivial homotopy group [16, 17, 34] of the vacua manifold  $\mathcal{M}$ . The  $n$ -th homotopy group  $\pi_n(\mathcal{M})$  is the set of homotopically equivalent classes of mappings from the  $n$ -sphere into the manifold  $\mathcal{M}$ . Domain walls exist when  $\pi_0(\mathcal{M}) \neq 0$  while  $\pi_2(\mathcal{M}) \neq 0$  means monopoles and  $\pi_3(\mathcal{M}) \neq 0$  textures. In the case of a string, and at large distances from it, the vacuum configuration of the scalar field, for a connected and simply connected  $\mathcal{G}$ , is given by

$$\phi(\theta) = g(\theta)\phi_0, \quad g(\theta) = e^{i\tau\theta} \quad (3.1)$$

where  $\tau$  is some generator of  $\mathcal{G}$  not in the algebra of  $\mathcal{H}$ ,  $\theta$  is the azimuthal angle measured around the string, and  $g(0)$  and  $g(2\pi)$  belong to two disconnected pieces of  $\mathcal{H}$ . The presence of strings is then signaled by the existence of noncontractible loops in the quotient space  $\mathcal{G}/\mathcal{H}$ . In what follows we analyze in a general scheme the TD arising in the spontaneous SB of the model  $[SU(6)]^3 \times Z_3$ . We will see that some interesting topological aspects of Lie groups are involved in the computation of TD.

Let  $\mathcal{H} \rightarrow \mathcal{G} \rightarrow \mathcal{G}/\mathcal{H}$  be a generic Lie group fibration which determines the spontaneous SB step at certain energy scale  $M_X$ . The associated homotopy

sequence to this fibration reads [35]

$$\dots \rightarrow \pi_n(\mathcal{G}) \rightarrow \pi_n(\mathcal{G}/\mathcal{H}) \rightarrow \pi_{n-1}(\mathcal{H}) \rightarrow \pi_{n-1}(\mathcal{G}) \rightarrow \dots$$

For  $n = 1$  we obtain when  $\mathcal{G}$  is connected ( $\pi_0(\mathcal{G}) = 0$ ) and simply connected ( $\pi_1(\mathcal{G}) = 0$ )

$$0 \rightarrow \pi_1(\mathcal{G}/\mathcal{H}) \rightarrow \pi_0(\mathcal{H}) \rightarrow 0 \quad (3.2)$$

or equivalently

$$\pi_1(\mathcal{G}/\mathcal{H}) \cong \pi_0(\mathcal{H}), \quad (3.3)$$

where “ $\cong$ ” must be read as isomorphic. Therefore, the only way to admit strings in whatever step of the SB is that the corresponding residual group  $\mathcal{H}$  must be not connected.

There are many possibilities for the chain of SB from  $G = [SU(6)]^3 \times Z_3$  down to  $SU(3)_c \otimes U(1)_{EM}$ . Nevertheless, most of them break  $G$  down to connected groups. An economical SB scheme

$$G \xrightarrow{M_R} G_1 \xrightarrow{M_H} G_{SM} \xrightarrow{M_L} G_r = SU(3)_c \otimes U(1)_{EM}, \quad (3.4)$$

where  $G_{MS}$  is the SM group and  $G_1 = SU(6)_L \otimes SU(4)_c \otimes SU(2)_c \otimes SU(4)_R \otimes SU(2)_R \otimes U(1)$  was analyzed in Ref. [36]. At each step however the residual symmetry group is connected and therefore no strings are formed. The mass scales have the hierarchy  $M_R \sim 10^{11} \text{ GeVs} > M_H \sim 10^8 \text{ GeVs} \gg M_L \sim 10^2 \text{ GeVs}$ , which are in a good agreement with those obtained from the analysis of the generational see-saw mechanism in the model [37]. The last pattern is implemented by three sets of Higgs fields in the irrep 675. Their respective vevs are displayed in the second reference in [36].

From here, the only way to break the symmetry down to a not connected group  $H$  is keeping the factor  $Z_3$  intact in the first step. Nevertheless, as we will show, there is only one way to do that and this SB scheme does not produce strings.

As we argued in the past section, we choose the Higgs content to be of the form  $Z_3\phi(\overline{n}, 1, n)$ , with  $n = 15$  or  $21$ . Hence, if we require that

$$G = [SU(6)]^3 \times Z_3 \rightarrow H, \quad (3.5)$$

where  $H$  contains the  $Z_3$  symmetry unbroken, then, in order to respect this symmetry, the tensor structure of  $\langle\phi\rangle$  must be the same in the  $L$ ,  $R$  and  $C$  spaces. Since ordinary color corresponds to  $\alpha = 1, 2, 3$  in the fundamental representation of  $SU(6)_c$ , the tensor indices of the terms in  $\langle\phi\rangle$  can be only 4, 5 or 6 in all the three spaces. On the other hand  $SU(2)_L$  should not be broken in the first step, that is  $\langle\phi\rangle$  must take the direction of the singlets of  $SU(2)_L$ ,

$$\begin{aligned} 21 & : \quad \{1, 4\} - \{2, 3\}, \quad \{1, 6\} - \{2, 5\}, \quad \{3, 6\} - \{4, 5\}, \\ 15 & : \quad [1, 4] - [2, 3], \quad [1, 6] - [2, 5] \quad [3, 6] - [4, 5], \\ & \quad [1, 2], \quad [3, 4], \quad [5, 6], \end{aligned}$$

since the  $SU(6)_L$  (and  $SU(6)_R$ ) indices are arranged according to the following scheme:

$$\begin{array}{ccccc} & & \leftarrow & SU(3) & \rightarrow \\ & & & & \\ \uparrow & 1 & 3 & 5 & \\ SU(2) & & & & \\ \downarrow & 2 & 4 & 6 & \end{array} \quad (3.6)$$

Therefore, the only Higgs field which maintains  $Z_3$  unbroken is  $\phi_0 \equiv \phi(675) = Z_3\phi(\overline{15}, 1, 15)$  with vevs in the direction  $[a, b], [\alpha, \beta], [A, B] = [5, 6]$ .

The symmetry breaking implemented by these vevs is (see appendix and Ref. [36])

$$G \xrightarrow{\langle\phi_0\rangle} [SU(4) \otimes SU(2)]^3 \otimes U(1)_\Sigma \times Z_3, \quad (3.7)$$

where  $[G']^3 \equiv G'_L \otimes G'_c \otimes G'_R$ ,  $G' = SU(4) \otimes SU(2)$ , and  $U(1)_\Sigma$  is generated by

$$T_\Sigma \equiv \frac{1}{\sqrt{3}} [T \otimes 1 \otimes 1 + 1 \otimes T \otimes 1 + 1 \otimes 1 \otimes T] \quad (3.8)$$

with

$$T = \text{diag}\{1, 1, 1, 1, -2, -2\}/\sqrt{6}. \quad (3.9)$$

Once we have fixed the first step of the SB pattern, the subsequent steps are very constrained. In fact the more economical set of Higgs fields which maintain this first step, and solve the rge in a proper way, uses six 675 to break  $H$  down to the SM in the following way (see the appendix)

$$G \xrightarrow{M_G} H \xrightarrow{M_R} I \xrightarrow{M_H} G_{SM} \xrightarrow{M_L} G_r \quad (3.10)$$

where  $M_X$  ( $X= G,R,H,L$ ) represent the mass scale of the corresponding SB step and  $I = SU(4)_L \otimes SU(2)_L \otimes SU(4)_c \otimes SU(2)_c \otimes Sp(4)_R \otimes SU(2)_R \otimes U(1)_\Sigma$ . The hierarchy of the mass scales  $M_G > M_R \geq M_H \gg M_L \sim 10^2 GeVs$ , is enough to cope with the experimental results. Precise values of  $M_G$ ,  $M_R$  and  $M_H$  are obtained from the rge (2.7 and 2.8). Figures 1 and 2 show the corresponding ones of  $M_G$  and  $M_R$  as a function of the values taken for  $M_H$ , when we use five or six 675 to make the breaking down to the SM. Also we plotted the identity function for  $M_H$ , looking for the limit point  $M_R = M_H$  where we have just an intermediate scale between  $M_G$  and  $M_L$ . As it can be noted, the intermediate scale is necessary to secure that  $M_G \leq 10^{19} GeVs$ , in the more economical case, but not in the other one.

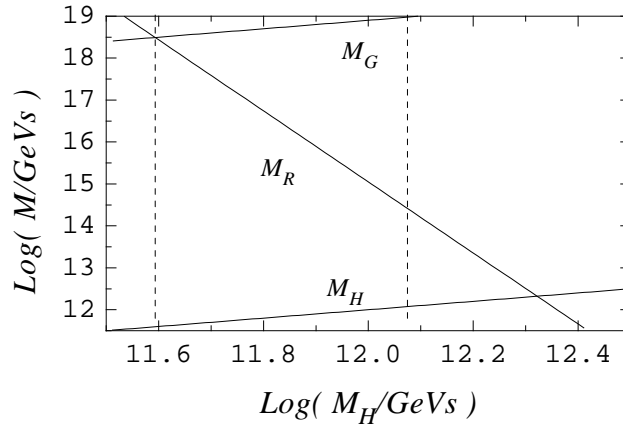


Figure 1: Evolution of the mass scales  $M_G$ ,  $M_R$  as a function of  $M_H$  when five 675 are used to break  $G$  down to the SM. Notice that the point  $M_R = M_H$  is out of the window  $M_H \leq M_R < M_G \leq 10^{19} GeVs$ .

Now,  $G$  in the present case is a not connected group due the factor  $Z_3$ , then the sequence (3.2) is not longer valid. Instead we have the complete homotopy sequence for  $n = 1$

$$\pi_1(G) \rightarrow \pi_1(G/H) \rightarrow \pi_0(H) \rightarrow \pi_0(G). \quad (3.11)$$

In other words, for the case considered above, we have

$$0 \rightarrow \pi_1(G/H) \rightarrow Z_3 \rightarrow Z_3.$$

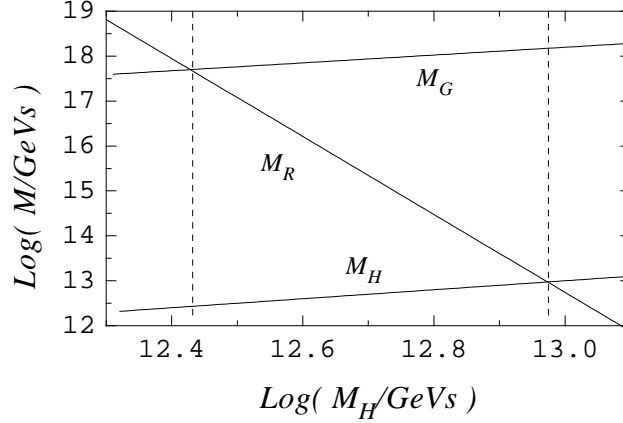


Figure 2: Development of the mass scales when we use six 675 for the SB. Now the point  $M_R = M_H$  is consistent with  $M_G \leq 10^{19} \text{ GeVs}$ .

Here we have used the usual relation  $\pi_i(X \times Y) = \pi_i(X) \oplus \pi_i(Y)$  for any  $X, Y$  topological spaces and the following theorem [35]:

Let  $Q$  be a discrete finite group of  $N$  elements, then  $\pi_n(Q) = 0$  for all  $n \neq 0$  and  $\pi_0(Q) \cong Z_N$ . Here  $Z_N$  is the finite cyclic group of order  $N$ .

In order to compute  $\pi_1(G/H)$  we would like first to explore the topology of the quotient manifold  $\mathcal{M} = G/H$ . It is easy to see that globally it looks like the product of two complex Grassmannians  $G_4(\mathbb{C}^6)$  and the quotient manifold  $G_4(\mathbb{C}^6)/U(1)_\Sigma$ . Thus the quotient  $\mathcal{M} = G/H$  looks topologically like

$$\mathcal{M} = G/H = G_4^L(\mathbb{C}^6) \times G_4^c(\mathbb{C}^6) \times G_4^R(\mathbb{C}^6)/U(1)_\Sigma. \quad (3.12)$$

Here  $G_4(\mathbb{C}^6)$  is represented as the homogeneous space

$$G_4(\mathbb{C}^6) = \frac{SU(6)}{SU(4) \times SU(2)}. \quad (3.13)$$

The general case is

$$G_k(\mathbb{C}^n) \equiv \frac{SU(n)}{SU(n-k) \times SU(k)}. \quad (3.14)$$

It is very well known [38] that the complex Grassmannian is a compact, connected and simply connected homogeneous space and the same occurs with a complex Grassmannian divided by any compact, connected and simply connected space (in our case it is  $U(1)_\Sigma$  which is topologically the circle). Thus we conclude that

$$\pi_1(\mathcal{M}) = 0. \quad (3.15)$$

and therefore there are not strings at the scale  $M_G$ . From the structure of the SB chain (3.10) the presence of strings might only occurs at the first SB step. We saw that it does not occurs and therefore there is no way to produce stable strings in the model. Nevertheless, metastable strings could be formed during a intermediate phase transition if the sequence

$$\mathcal{G}' \subset \mathcal{G} \rightarrow \mathcal{G}'' \otimes U(1) \otimes U(1)' \rightarrow \mathcal{G}'' \otimes U(1) \quad (3.16)$$

could be implemented in the model [17, 39]. In this case the strings connecting monopole-antimonopole pairs are associated to the broken  $U(1)'$  symmetry and therefore they might decay by nucleation of pairs [40] giving a possible contribution to the HECR events.

Effectively, a sequence of the type (3.16) may be implemented in the model as it follows from the analysis done in reference [29, 36]

$$G \xrightarrow{M_R} G_1 \xrightarrow{M_H} G_{SM} \otimes U(1)' \xrightarrow{M_S} G_{SM} \xrightarrow{M_L} G_r. \quad (3.17)$$

In the sequence (3.17) the mass scales have the hierarchy  $M_R \sim 10^{12} \text{ GeV} > M_H \sim 10^8 \text{ GeV} > M_S > M_L$ . The scale  $M_S$  corresponds to the phase transition at which metastable strings are formed.  $M_S$  is not fixed by the rge since the scalar fields which breaks the  $U(1)'$  symmetry are singlets of the SM group.

On the other hand, the presence of domain walls formed during the phase transitions of a universe with the gauge symmetry  $\mathcal{G}$  is a cosmological problem

which can not be solved, then, a realistic model should not predict them in order to be considered as admissible. In our case, we have to compute the homotopy group which determines their presence. That means to compute  $\pi_0(\mathcal{M})$  for all the steps of the SB chains.

A second consequence from the topological structure of  $\mathcal{M}$  Eq. (3.12) is that

$$\pi_0(\mathcal{M}) = 0. \quad (3.18)$$

Then there are no domain walls at the  $M_R$  scale.

For the second step  $H \rightarrow I$ , the only broken symmetry is  $SU(4)_R$  whose residual symmetry is  $Sp(4)_R$ , so, we compute easily  $\pi_0(H/I)$  to be trivial, and conclude that again there are no domain walls formed in this step. This is because  $SU(4)$  and  $Sp(4)$  are connected, simply connected and compact Lie groups. In the next step,  $I \rightarrow G_{SM}$ , the calculation gives  $\pi_0(I/G_{SM}) = 0$ , and finally the breaking of the SM symmetry does not produce domain walls. Hence, they are absent in the chain (3.10).

The same analysis for the minimal chain (3.4) gives

$$\pi_0(G/G_1) = \pi_0(G_1/G_{SM}) = 0, \quad (3.19)$$

and in a similar way from sequence (3.17),  $\pi_0(G_1/G_{SM} \times U(1)') = \pi_0(G_{SM} \times U(1)'/G_{SM}) = 0$ , because all involved groups are compact. So, we can conclude that the model is free of domain walls. Notice that in all computations of the above homotopy groups we have used the standard techniques of semistable homotopy groups of Ref. [35].

## 4 Monopoles in $[SU(6)]^3 \times Z_3$ .

Monopoles are the more common kind of topological defects in GUM's. As we mentioned above, they are associated with the existence of a non trivial homotopy group:  $\pi_2(\mathcal{M}) \neq 0$  [16, 17, 34]. Then, the presence of  $U(1)$  factors in the residual symmetry after the breaking is sufficient to show the formation at that step of monopoles in the model, through the Kibble mechanism [10].

By construction, all the GUM's produce monopoles, even when they break the unified symmetry to the SM group in a simple step, because the SM group contains a  $U(1)_Y$  factor associated to the hypercharge. Then if the conventional philosophy of the GUT's is correct, monopoles must exist unless there is some unknown mechanism which kills them. Of course an obvious way to relax this problem is including a  $U(1)_Y$  factor at the level of the GUT, but in this case the model has not a simple gauge coupling in a natural way. This is a motivation to consider monopoles as a good candidates to be sources of the HECR [7, 11, 12, 14, 24]. Monopolonium [41] is a possible bound state formed from a monopole-antimonopole pair, which spirals in and finally collapses. This is a very slow process which fails in solving the monopole overabundance of the early universe. Nevertheless, if by some mechanism the universe is never monopole dominated, or also, if they exist at the level of abundance compatible with known experimental [21] and phenomenological upper bounds [15], then the late annihilation of monopoles could be precisely the mechanism that we need from the point of view of generating HECR, which must be produced only in the contemporary cosmic epoch. Besides the associated symmetry, the only clear difference between monopoles produced by a specific GUM with respect to another one is the mass scale at which they appear, because it depends strongly in the SB pattern. In most of the models which achieve the simple unification, the scale is of the order of  $10^{16} \text{ GeVs}$ . If this is the case, the production of HECR based on monopole acceleration must be ruled out, because of the upper bound for the monopole mass  $M \leq 10^{10\pm1} \text{ GeVs}$  [14] obtained from the phenomenological and experimental bounds to their flux [15, 21].

In the case we are considering here, the minimal pattern of the SB given by (3.4) produces monopoles at all the scales, Then the more massive ones should appear around  $M_R \sim 10^{11} \text{ GeVs}$ . Also monopoles are formed at the mass scales  $M_H \sim 10^8 \text{ GeVs}$  and  $M_L \sim 10^2 \text{ GeVs}$ . Now, monopole annihilation could not give an important contribution to the HECR in this model, but instead the acceleration mechanism could be accepted, even if the mass of the  $10^{11} \text{ GeVs}$  monopoles get in conflict with the experimental data, because, in such a case one can invoke inflation below or at that scale. In this case, the massive monopoles should have no more relevance for the HECR, but the lighter ones should produce HECR by the acceleration mechanism. In a similar way, in the pattern given by (3.17) monopoles appear at the same

scales, but now two classes of them are produced at the scale  $M_H$ . Those associated to the broken symmetry  $U(1)'$  become attached to strings as we argued in the above section. The other monopoles should be free and they could be accelerated to relativistic energies just like those produced in the pattern (3.4).

On the other hand, the exotic chain (3.10), has a higher unification scale ( $M_G \sim 10^{18} \text{ GeVs}$ ) than the more usual patterns, and also than the minimal chain in (3.4). For the model with this SB scheme

$$\pi_2(G/H) \cong Z. \quad (4.1)$$

Then we have supermassive monopoles at the same scale  $M_G$ . Now, the monopoles produced by this phase transition may give an important contribution to the HECR via the monopole-antimonopole annihilation mechanism. As we discuss before, this kind of monopoles could overclose the early universe, and therefore they constitute a serious cosmological problem. even so, as Sigl *et. al.* have argued [7, 11, 12], from numerical simulations, it looks that it is possible (with proper assumptions) that they could give a substantial contribution to the HECR flux. Notice that the chain (3.10) is the only known in this model with a unification scale as high as the one of a typical GUM. Hence, this SB pattern should be ruled out if the monopolonium annihilation mechanism is not confirmed by future observations like those planed in the “Pierre Auger Project”, which expects to collect six thousand events per year above  $10^{19} \text{ GeVs}$  in a  $6,000 \text{ Km}^2$  detector [42].

At the second step of the chain (3.10) the  $U(1)_\Sigma$  factor stays intact and not extra  $U(1)$  factors arise, then  $\pi_2(H/I) = 0$ , as the homotopy computation shows. Hence, no monopoles are formed at that step, The nexts steps produce residual  $U(1)$  symmetries. Then, as an easy calculations indicates, we have monopoles at the scales  $M_H$  and  $M_L$ , for this SB scheme.

Finally, we have to mention that local textures are formed from the phase transitions of the model in both schemes [Eq (3.4) and (3.10)]. We compute  $\pi_3(\mathcal{M})$  for all the steps and conclude that local textures appear only at the scales  $M_H$  and  $M_L$  in both SB patterns. However, their possible relation with the sources of the HECR is not clear until now.

## 5 Concluding remarks.

In this paper we have started the study of the TD arising in the model proposed in Refs. [27, 28, 29]. We have explored the possibility of breaking  $[SU(6)]^3 \times Z_3$  down to the SM group through a chain that would give rise to the formation of stable strings, however our conclusion is that it is not possible. Nevertheless, in certain patterns [Eqs. (3.16) and (3.17)] the model does admit metastable strings which connect monopole-antimonopole pairs. Basically, we have established that this vacuum configuration could appear at an scale below  $M_H \sim 10^8$  GeV. The decaying processes which involve these kind of hybrid defects could give a relevant contribution to the HECDR flux. Also, the homotopy analysis of the minimal scheme (3.4) of the SB chain and of the exotic one (3.10) shows that domain walls are absent, which is in very good agreement with cosmological restrictions. The same is valid in the case of the pattern (3.17) which produces metastable strings. Hence, this result could give us an indirect test of the viability of the model, besides its special properties mentioned along the paper. Also, the model contains local textures, but they are not of our interest now, because there is not a clear mechanism that involves them as sources of the HECDR.

Other TD relevant for the HECDR problem produced in the model are monopoles, whose energy scale depends strongly on the specific pattern chosen for the SB. We have analyzed here two schemes. While the minimal chain produces monopoles at all the relevant energy scales  $M_R \sim 10^{11}$  GeVs  $>$   $M_H \sim 10^8$  GeVs  $\gg$   $M_L \sim 10^2$  GeVs, in the new chain (3.10) they are formed at higher energies,  $M_G \sim 10^{18}$  GeVs and  $M_H \sim 10^{12.5}$  GeVs, and of course also at the SM scale  $M_L$ . As a matter of fact, if the annihilation mechanism is correct and it is confirmed by future data, then the model does not have any advantage over other GUM's because most of them predict the formation of monopoles at or above  $10^{16}$  GeVs, and then only a very precise analysis could rule out any of these models. On the other hand, the presence of monopoles in the more natural SB scheme given by (3.4) at not so high energies, could actually give to the model a real advantage over other GUM's in the case that the HECDR should be produced by the acceleration mechanism [14]. This is by the moment, an open problem to be analyzed in the future. Another open problem is the relation of the results obtained in

this paper and the recent measurements of the extragalactic diffuse gamma ray background [43] which also seems to be difficult to explain otherwise in terms of emissions from astrophysical objects. This problem has been recently studied in the context of supersymmetric GUT's [44], where strings formed at one scale below  $10^{14}$  GeV could simultaneously be source of high energy particles (with masses of the order of the breaking scale) and low energy higgs particles (with masses as low as 1 TeV). In these scenarios, while the high energy particles may contribute to the HECR flux, the low energy higgs' decay can account for the extragalactic diffuse gamma ray background. However, in a non supersymmetric GUT, the possibility of implementing this mechanism is still an open question. In supersymmetric theories the basic ingredient are the flat directions of the scalar potential, which is absent in the non supersymmetric theories. However, the formation of hybrid defects at low energy scales, as in the case of the chain (3.17) where scalars could appear at low energy may give such contributions, but more analysis is required to establish this possibility.

Finally the TD discussed along the paper for the chains (3.4) and (3.10) are not metastable but topologically stable. This is due to the fact that they do not satisfy the Preskill-Vilenkin criterion [39].

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## Appendix A

In this appendix we give some details about the SB implemented in section 3. First, since  $\langle\phi_0\rangle$  is  $Z_3$ -invariant, it induces the breaking

$$G \rightarrow G' \times Z_3$$

where  $G'$  is a  $Z_3$ -invariant subgroup of  $[SU(6)]^3$ .

Next, consider the group  $SU(4) \otimes SU(2) \subset SU(6)$ , where  $SU(4)$  ( $SU(2)$ ) acts only on the indices 1, 2, 3, 4 (5, 6) of the fundamental representation of  $SU(6)$ .  $SU(4)$  ( $SU(2)$ ) is the largest simple group contained in  $SU(6)$  which acts only on the subspace with tensor indices 1, 2, 3, 4 (5, 6). For this decomposition of the space, the branching rules read

$$\begin{aligned} SU(6) &\rightarrow SU(4) \otimes SU(2) \\ 6 &\rightarrow (4, 1) + (1, 2) \\ 15 &\rightarrow (1, 1) + (6, 1) + (4, 2) \\ 21 &\rightarrow (10, 1) + (1, 3) + (4, 2) \\ 35 &\rightarrow (1, 1) + (15, 1) + (1, 3) + (4, 2) + (\bar{4}, 2), \end{aligned}$$

which shows that there is only one component in the irrep 15 of  $SU(6)$  which is singlet of  $SU(4) \otimes SU(2)$  and that it corresponds precisely to the direction [5, 6], which is the direction in the  $L$ ,  $R$ , and  $c$  spaces where we have demanded  $\langle\phi_0\rangle \neq 0$ . On the other hand the maximal subgroup of  $SU(6)$  which contains  $SU(4) \otimes SU(2)$  is  $SU(4) \otimes SU(2) \otimes U(1)$  where  $U(1)$  is generated by the matrix displayed in eq. (3.9). Obviously these  $U(1)$  factors in  $G$  are broken by  $\langle\phi_0\rangle$ , but the sum of their generators,  $T_\Sigma$ , given in eq. (3.8) satisfies

$$T_\Sigma \langle\phi_0\rangle = 0$$

and therefore  $U(1)_\Sigma$  generated by  $T_\Sigma$  remains unbroken. Therefore, the SB induced by  $\langle\phi_0\rangle$  is

$$G \rightarrow [SU(4) \otimes SU(2)]^3 \otimes U(1)_\Sigma \times Z_3.$$

It is important to note, that now in this chain, the ordinary color group  $SU(3)_c \subset SU(4)_c$ , and the the left group  $SU(4)_L \otimes SU(2)_L$  decomposes in

$SU(2)_L \otimes SU(2)_{HL}$ , where the last  $SU(2)_L$  is the standard electroweak group and the horizontal group  $SU(2)_{HL}$  acts only in the space of the the first two families, as it follows from the branching rules

$$\begin{aligned} SU(4)_L \otimes SU(2)_L &\rightarrow SU(2)_L \otimes SU(2)_{HL} \\ (4, 1) &\rightarrow (2, 2) \\ (1, 2) &\rightarrow (2, 1). \end{aligned}$$

The subsequent steps of the SB were chosen in order to get the highest contribution from the Higgs sector to the rge. The vevs of the Higgs fields used for next steps of the SB are

$$\langle \phi_{1[a,b]}^{[A,B]} \rangle = \langle \phi'_{1[a,b]}^{[A,B]} \rangle = \langle \phi_{2[a,b]}^{[A,B]} \rangle = 0,$$

and

$$\begin{aligned} \langle \phi_{1[\alpha,\beta]}^{[a,b]} \rangle = \langle \phi_{1[A,B]}^{[\alpha,\beta]} \rangle = M_H &\quad \text{for } [a, b], [A, B] = [1, 2] = [4, 5] = -[3, 6], \\ &\quad \text{and } [\alpha, \beta] = [4, 5], \end{aligned}$$

$$\begin{aligned} \langle \phi'_{1[\alpha,\beta]}^{[a,b]} \rangle = \langle \phi'_{1[A,B]}^{[\alpha,\beta]} \rangle = M_H &\quad \text{for } [a, b], [A, B] = -[1, 2] = [4, 5] = -[3, 6], \\ &\quad \text{and } [\alpha, \beta] = [4, 5], \end{aligned}$$

$$\begin{aligned} \langle \phi_{2[\alpha,\beta]}^{[a,b]} \rangle = \langle \phi_{2[A,B]}^{[\alpha,\beta]} \rangle = M_H &\quad \text{for } [a, b], [A, B] = -[2, 5] = [1, 6] = [3, 4], \\ &\quad \text{and } [\alpha, \beta] = [4, 5], \end{aligned}$$

and finally

$$\begin{aligned} \langle \phi'_{2[\alpha,\beta]}^{[a,b]} \rangle = M_H &\quad \text{for } [a, b] = -[2, 5] = [1, 6] = -[3, 4], \text{ and } [\alpha, \beta] = [4, 5], \\ \langle \phi'_{2[A,B]}^{[\alpha,\beta]} \rangle = M_H &\quad \text{for } [A, B] = [2, 6] \text{ and } [\alpha, \beta] = [4, 6], \\ \langle \phi'_{2[a,b]}^{[A,B]} \rangle = M_R &\quad \text{for } [A, B] = [2, 3] = -[1, 4] \text{ and } [a, b] = [5, 6]. \end{aligned}$$

They, together with  $\langle \phi_4 \rangle$ , are sufficient to implement the SB (3.10). We must however add one or more Higgs fields, in order to solve consistently the rge. Those fields are chosen to take vevs along the directions of the  $(4, 2) + (6, 1)$  irrep of  $SU(4)_{L,R} \otimes SU(2)_{L,R}$ , and the  $(4, 2)$  of  $SU(4)_c \otimes SU(2)_c$  in such a way that  $\langle \phi_{[a,b]}^{[A,B]} \rangle = 0$ .

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